Section 5.4

Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus (PART 1)

Evaluating Definite Integrals

If f is continous on [a,b], then the function

$$F(x) = \int_a^b f(t)\,dt$$

has a derivative at every point x in [a,b] and

$$\frac{dF}{dx} = \frac{d}{dt}\int_a^x f(t)\,dt = f(x)$$

Find

$$\frac{d}{dx}\int_{-\pi}^{x}\cos t\,dt \qquad \text{and} \qquad \frac{d}{dx}\int_{0}^{x}\frac{1}{1+t^2}\,dt$$

More Examples

Find g'(x)

a.) $g(x) = \int_{1}^{x} (t^2 + 1)^3 \, dt$

b.) $g(x) = \int_{x}^{2} \cos(t^2 + t) \, dt$

Find dy/dx

$$y = \int_{1}^{x^2} \cos(t)\, dt$$

The Fundamental Theorem with the Chain Rule

Find g'(x)

$$g(x) = \int_{1}^{\sqrt{x}} \frac{a^2}{a^2 + 1}\, da$$

Find dy/dx

a.) $y = \int_{x}^{5} 3t \sin t \, dt$

b.) $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} \, dt$

$$\frac{d}{dx}\int_x^7 \sqrt{\sin t}\,dt$$

$$f(x) = -\sqrt{\sin x}$$

$$\frac{d}{dx}\int_{x}^{e^x}\frac{t^2+1}{2t}\,dt$$

$$= \frac{d}{dx}\int_0^{e^x}\frac{t^2+1}{2t}\,dt - \frac{d}{dx}\int_0^{x}\frac{t^2+1}{2t}\,dt$$

$u = e^x \qquad \frac{du}{dx} = e^x$

$$= \frac{d}{dx}\int_0^{u}\frac{t^2+1}{2t}\,dt$$

$$= \frac{u^2+1}{2u}\cdot\frac{du}{dx}$$

$$= \frac{e^{2x}+1}{2\cancel{e^x}}\cdot \cancel{e^x}$$

$$-\ \frac{x^2+1}{2x}$$

$$\frac{x^2+1}{2x} = \frac{e^{2x}+1}{2} - \frac{x^2+1}{2x}$$

$$\frac{d}{dx}\int_4^x \sqrt{\sin t^3 - 4t}\; dt$$

$$f(x) = \sqrt{\sin x^3 - 4x}$$

$$\frac{d}{dx}\int_2^{\cos x}\sqrt{t}\,dt$$

$u = \cos x \qquad \frac{du}{dx} = -\sin x$

$$\frac{d}{dx}\int_2^{u}\sqrt{t}\,dt = \sqrt{u}\,\frac{du}{dx}$$

$$= \sqrt{\cos x}\cdot -\sin x$$

$$= -\sin x\sqrt{\cos x}$$

$$\frac{d}{dx}\int_{3x}^{5} t^3 + 1\, dt = \frac{d}{dx} - \int_{5}^{3x} t^3 + 1\, dt.$$

$u = 3x \quad \frac{du}{dx} = 3$

$$= \frac{d}{dx}\left(-\int_{5}^{u} t^3 + 1\, dt\right)$$

$$= -\left(u^3 + 1\right)\frac{du}{dx}$$

$$= -3\left((3x)^3 + 1\right) = -3\left(27x^3 + 1\right)$$

$$\frac{d}{dx} \int_{x}^{2x^2} \sin t \, dt$$

Find f'(x)

$$f(x) = \int_2^x \sin t^2 \, dt$$

$$f'(x) = \frac{d}{dx}\int_2^x \sin t^2 \, dt$$

$$f'(x) = \sin x^2$$

Find the derivative

$$g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\, dt$$

$$g'(x) = \frac{2x}{\sqrt{2+x^8}} - \frac{\sec^2 x}{\sqrt{2+\tan^4 x}}$$

Find a function y = f(x) with a derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition f(3) = 5

when $x = 3$

$y = 5$

$$f(x) = \int_3^x \tan t \, dt + 5$$

$$y = \int_3^3 \tan t \, dt = 0$$

Find a function f(x) that has a derivative of

$$\frac{\sin x}{\sqrt{x}}$$

and passes through the point (7,-2)

$$f(x) = \int_7^x \frac{\sin t}{\sqrt{t}}\,dt - 2$$

Find a function f(x) that has a derivative

$$\sqrt{\cos x}$$

and passes through the point $(-\pi, 4)$

$$f(x) = \int_{-\pi}^{x} \sqrt{\cos t}\, dt + 4$$

Find a function with a derivative of $2x^2 + 4x$ that passes through (2,9)

$$f(x) = \int_2^x 2t^2 + 4t \, dt + 9$$

$$= \left. \frac{2t^3}{3} + \frac{4t^2}{2} \right|_2^x + 9$$

$$= \left. \frac{2t^3}{3} + 2t^2 \right|_2^x + 9$$

Evaluate $\displaystyle\int_{-1}^{3} (x^3 + 1)\,dx$

$$= \left. \frac{x^4}{4} + x \right|_{-1}^{3}$$

$$= \left(\frac{3^4}{4} + 3\right) - \left(\frac{(-1)^4}{4} - 1\right)$$

$$= \left(\frac{81}{4} + 3\right) - \left(\frac{1}{4} - 1\right) =$$

Fundamental Theorem of Calculus (PART 2)

If f is continuous at every point of [a,b], and if F is any antiderivative of f on [a,b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Also called the INTEGRAL EVALUATION THEOREM

SUBSTITUTE!!!

Find the area of the region between the curve $y = 4 - x^2$, $0 \leqslant x \leqslant 3$, and the x axis

Total

Area

$$\int_0^3 (4 - x^2)\,dx =$$

How to Find the Total Area Analytically

To find the area between the graph of y = f(x) and the x - axis over the interval [a,b] analytically,

1.) Partition [a,b] with the zeros of f
2.) Integrate f over each subinterval
3.) Add the absolute values of the integrals

Find the area of the region between the curve $y = x \cos 2x$ and the x-axis over the interval $-3 \le x \le 3$.

Antiderivatives...

$$\int 3x^2\,dx = \frac{3x^3}{3} + C = x^3 + C$$

$$\int 2x^3\,dx = \frac{2x^4}{4} + C = \frac{x^4}{2} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2(2x)\,dx = \frac{\tan(2x)}{2} + C$$

$$\int e^x\,dx = e^x + C$$

$$\sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\cancel{\frac{3}{2}}} + C = \frac{2x^{\frac{3}{2}}}{3} + C$$

$$\frac{1}{x+1}\,dx = \ln(x+1) + C$$

$$\frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + C$$

$$\frac{2}{x^3}\,dx = \int 2x^{-3}\,dx = \frac{\cancel{2}x^{-2}}{-\cancel{2}} + C$$

$$= \frac{-1}{x^2} + C$$

Find the total area under the graph of $y = -x^2 + 4$ between $x = 0$ and $x = 5$

$$\int_{1}^{5} 2x^2 + x - 2 \, dx$$

$$\int_{-\pi}^{\frac{\pi}{2}} \cos x \, dx$$

$$\int_0^4 2e^x \, dx =$$

Evaluate $\displaystyle\int_{2}^{x} t^2 + 4t - 3 \, dt$

$$\frac{d}{dx}\ x^3 + 2x^2 - 3x - \frac{14}{3}$$

$$\int_0^{x^2} \cos t \, dt =$$

If $\displaystyle\int_0^x f(t)\,dt = x\cos(\pi x)$

Find f(4)

Suppose $\int_1^x g(t)\,dt = 2x^2 - 3x + 1$

Find $g'(x)$

If $f(x) = \int_a^x e^{t^2}\, dt$

If f(0) = 5. Find f(3)

$$F(x) = \int_{2}^{x} t^2 + 4t \, dt$$

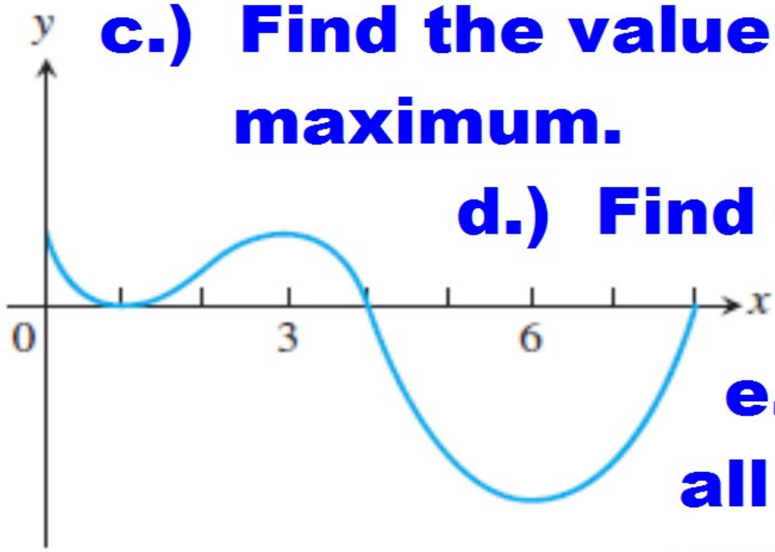
Find the equation of the tangent to F(x) at x = 5

The graph of a continuous function f with domain [0,8] is shown below. Let h be the function defined by $h(x) = \displaystyle\int_1^x f(t)\,dt$

a.) Find h(1)

b.) Is h(0) positive or negative?

c.) Find the value of x for which h(x) is a maximum.



d.) Find the value of x for which h(x) is a minimum.

e.) Find the x-coordinates of all points of inflections of the graph of y = h(x).

$$f(x) = \int_{2}^{x} \sin t^2 \, dt$$

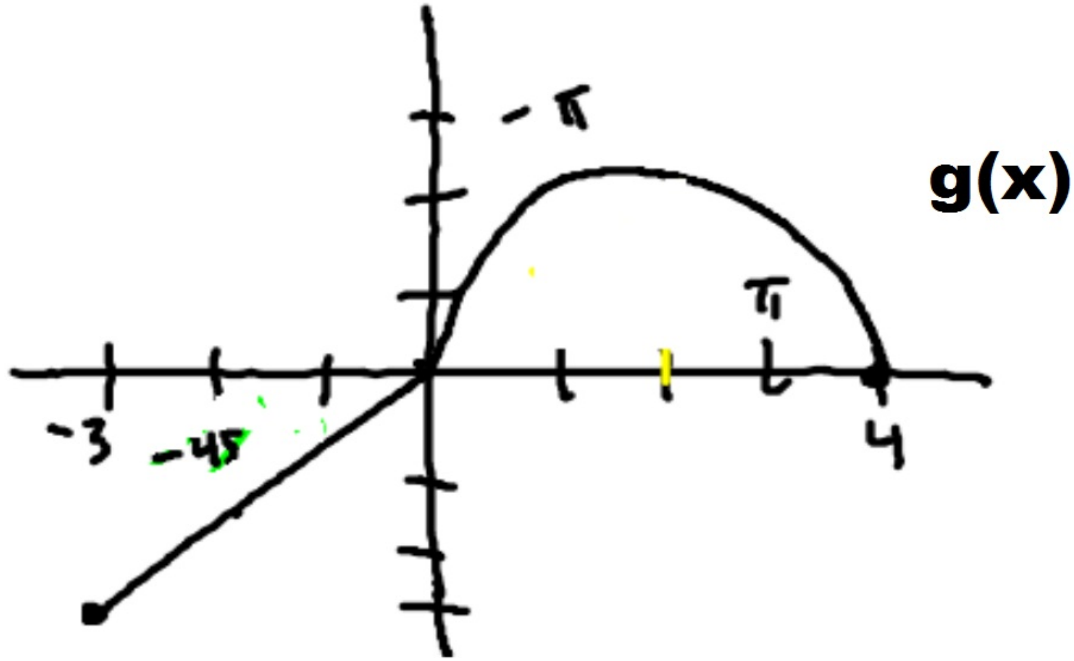
$[-\pi,\pi]$

When is f(x) increasing?

Maximum?

Is f(x) concave up or concave down?

POI?



1.) **Find H(0) =**

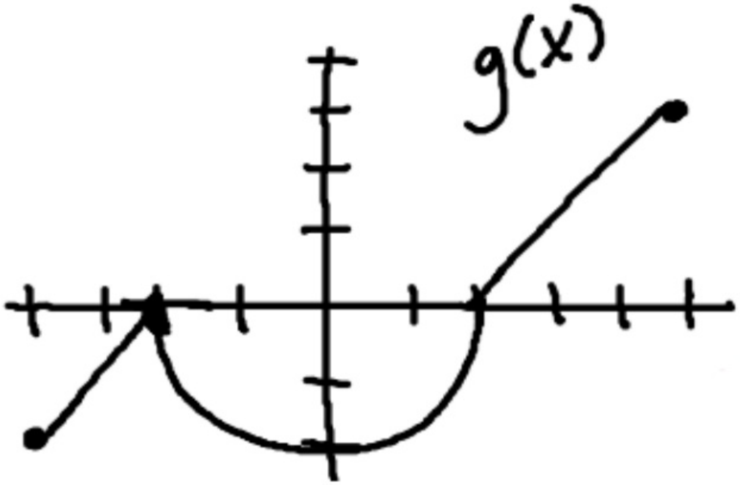
2.) **When is H(x) increasing?**

3.) **When does H(x) have a minimum?**

4.) **Is x = -3 or x = 4 the absolute max?**

5.) **Find H" (-2)**

6.) **H'(-1) =**



Let $H(x) = \int_2^x g(t)\,dt$

1.) Find H(2), H(0), H(5)

2.) Where is H(x) increasin concave down?

3.) Where does H(x) have an absolute maximum?

4.) Where does H(x) have point(s) of inflection?

5.) Find H'(0)

6.) Find H"(3)

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